TABLE II
CALCULATED RESULTS FOR THE DEVICE IN [13].

f	[GHz]	4	8
X_{s_m}	[Ω]	73.76	18.96
$R_{n_{min}}$	[Ω]	13.25	27.21
X_{s_M}	[Ω]	−116.51	−358.19
$R_{n_{max}}$	[Ω]	218.54	56.17
$R_{n_{sat}}$	[Ω]	145.89	55.96

To demonstrate experimental evidence for the validity of the analysis presented above, the results in [13] are considered, which show a minimum in R_n [13, Fig. 4]. If device parameters [13, page 324] are entered into (6), (7), and (8), the values of Table II are obtained in agreement with those results. The maximum in R_n is missing in [13, Fig. 4], since it occurs for a very large value of $(-X_s)$, where $R_n \simeq R_{n_{\max}} \simeq R_{n_{\text{sat}}}$.

VI. CONCLUSION

Closed-form expressions have been presented for the noise parameters with parallel and series feedback. It has been demonstrated that R_n always reaches a maximum and minimum, and the possibility of $R_n = 0$ has been pointed out. The same conclusions can be applied to g_n, since a duality principle exists. The theory shows that a minimum in the noise parameter R_n or g_n of either an active or passive black box may exist as long as its signal matrix is not purely real. A previous paper and its results have been used in order to demonstrate experimental evidence for the correctness of the formulas presented. This theory may help to design very low noise-feedback microwave amplifiers.

ACKNOWLEDGMENT

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Investigating Nonlinear Propagation in Dielectric Slab Waveguides

Jian-Guo Ma

***Abstract*—A numerical method is employed to analyze the TE-wave propagation in Kerr-like nonlinear dielectric waveguides in which a nonlinear film is sandwiched between two linear media. The dispersion curves dependent on the magnitude of the electric field are obtained. All the results can be used in future investigations of devices composed of nonlinear dielectric slab structures.**

***Index Terms*—Dispersion, Kerr-like, nonlinearity, waveguide.**

I. INTRODUCTION

It has been apparent for a long time that nonlinear propagation in optical and millimetric waveguides holds promise in the context of integrated signal processing [1]. In recent years, with the development of technology, guided waves in nonlinear dielectric slab waveguides received considerable attention owing to their potential applications to optical communications and optical computing.

For the nonlinear core waveguide, a general dispersion equation was developed in [2], using the modulus of a Jacobian elliptic function; however, spurious roots then appear in the dispersion equations [4]. The phase-plane approach was recently used in [1] to discuss the problem, which provides a physical interpretation of the results. This method can be applied to arbitrary nonlinearities. In all other cases, numerical methods such as in [3], [7], and [8], along with many others, have been employed.

In this paper, another numerical method is used to solve the nonlinear propagation in slab guides with a nonlinear core. The method transmits the values of the field from one boundary to another, therefore, it is called the transfer matrix method (TMM). In [9], the same idea was successfully used to numerically analyze the nonlinear planar waveguide with a linear core—a linear film is supported by a linear medium and covered by a nonlinear medium. In this paper, global coordinates are used to simplify the problem.

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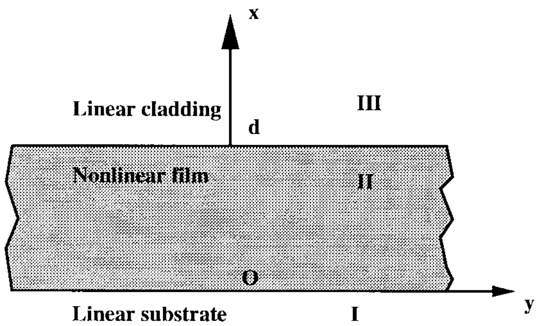

Fig. 1. A slab guide with a nonlinear core, a linear cladding, and a substrate.

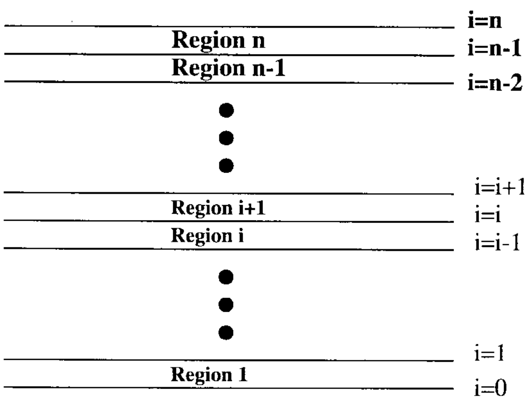

Fig. 2. Dividing the nonlinear Region II into n sub-regions.

II. Numerical Method

The schematic drawing of a three-layered slab guide with a Kerr-like nonlinear guiding film bounded by linear media is shown in Fig. 1. Restricting ourselves to TE waves, only the y-component of the electric field is nonzero, the electric field E_y, propagating along the z axis $e^{-j(\beta z-\omega t)}$, must satisfy the following equation in Region II for Kerr-like nonlinearity [1]–[8] with the nonlinear coefficient α:

$$\frac{d^2E_y}{dx^2}+k_o^2(\epsilon_{r2}-\beta^2+\alpha|E_y|^2)E_y=0, \quad 0\geq x\geq d \tag{1}$$

and in Regions I and III:

$$\frac{d^2E_y}{dx^2}-k_o^2(\beta^2-\epsilon_{r1})E_y=0, \qquad x\leq 0 \tag{2}$$

$$\frac{d^2E_y}{dx^2}-k_o^2(\beta^2-\epsilon_{r3})E_y=0, \qquad x\geq d. \tag{3}$$

Here ϵ_{r2} is the linear part of the dielectric constant of Region II, and ϵ_{ri} $(i=1,3)$ is the dielectric constant in Regions I and III, respectively. Solutions in the linear regions are [1], [7]

$$E_y=\begin{cases} E_oe^{k_cx}, & x\leq 0; \quad k_c^2=\beta^2-k_o^2\epsilon_{r1}\\ \overline{E}_oe^{-\overline{k}_c(x-d)}, & x\geq d; \quad \overline{k}_c^2=\beta^2-k_o^2\epsilon_{r3}.\end{cases} \tag{4}$$

For solving (1) Region II is divided into n sub-regions (as shown in Fig. 2) and in each sub-region $x\in[x_{i-1},x_i]$ $(i=1,2,\cdots,n)$ (1) is valid. Then using the value of the field at $x=x_{i-1}, E_y(x_{i-1})$ replaces $E_y(x)$ in the term $\alpha|E_y(x)|^2$. Now (1) is linearized to be reduced to a linear equation in each sub-region $[x_{i-1},x_i]$, approximately. Its solution is like that in a linear dielectric slab guide. Let

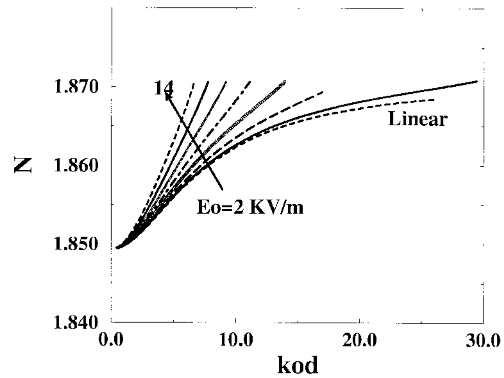

Fig. 3. The dispersion curve of the first TE-mode, TE_0 mode. $N=\beta/k_o$ $(\epsilon_{r1}=\epsilon_{r3}=3.42, \epsilon_{r2}=3.5$ and $\alpha>0)$.

$E_i(x)=E_y(x)$, $x\in[x_{i-1},x_i]$ $(i=1,2,\cdots,n)$:

$$E_i(x)=A_i\sin(k_ix)+B_i\cos(k_ix) \tag{5}$$

with

$$k_i^2=k_o^2(\epsilon_{r2}-\beta^2+\alpha|E_i(x_{i-1})|^2), \quad i=1,2,\cdots,n. \tag{6}$$

From the Maxwell's equations it is known that using the boundary conditions at $x=x_{i-1}$ and $x=x_i$, the field values at the boundaries can be connected as follows:

$$\begin{bmatrix} E_i(x_i)\\ \dfrac{dE_i(x_i)}{dx}\end{bmatrix}=[M_i(x_i)][M_i(x_{i-1})]^{-1}\begin{bmatrix} E_{i-1}(x_{i-1})\\ \dfrac{dE_{i-1}(x_{i-1})}{dx}\end{bmatrix} \tag{7}$$

with

$$M_i(x_{i-1})=\begin{bmatrix}\sin(k_ix_{i-1}) & \cos(k_ix_{i-1})\\ k_i\cos(k_ix_{i-1}) & -k_i\sin(k_ix_{i-1})\end{bmatrix}, \quad i=1,2,\cdots,n. \tag{8}$$

Using (8) repeatedly, the coefficients in the first region A_1, B_1 will be connected to the coefficients in the region n:

$$\begin{bmatrix} E_n(x_n)\\ \dfrac{dE_n(x_n)}{dx}\end{bmatrix}=\left\{\prod_{i=2}^{n}[M_i(x_i)][M_i(x_{i-1})]^{-1}\right\}\begin{bmatrix} E_2(x_1)\\ \dfrac{dE_2(x_1)}{dx}\end{bmatrix} =\left\{\prod_{i=2}^{n}[M_i(x_i)][M_i(x_{i-1})]^{-1}\right\}[M_1(x_1)]\cdot\begin{bmatrix}A_1\\ B_1\end{bmatrix}. \tag{9}$$

Using the boundary conditions at $x=x_o=0$ and $x=x_n=d$, the dispersion equation can be obtained:

$$\overline{E}_o\begin{bmatrix}1\\ -\overline{k}_c\end{bmatrix}=\left\{\prod_{i=2}^{n}[M_i(x_i)][M_i(x_{i-1})]^{-1}\right\}\begin{bmatrix}\dfrac{k_c}{k_1}\\ 1\end{bmatrix}E_o. \tag{10}$$

III. Results

As an example, a nonlinear dielectric slab guide is studied with the parameters: $\epsilon_{r1}=\epsilon_{r3}=3.42$, $\epsilon_{r2}=3.5$, $d=1\,\mu$m, and $\alpha=\pm1.625\times10^{-10}\,(\text{m/V})^2$ for focusing and defocusing nonlinearity, respectively. Letting $N=\beta/k_o$ be the effective index of the guide, the dispersion curves for TE_0 and TE_1 modes dependent on the magnitude of the field at $x=0$, E_o, are given in Figs. 3 and 4,

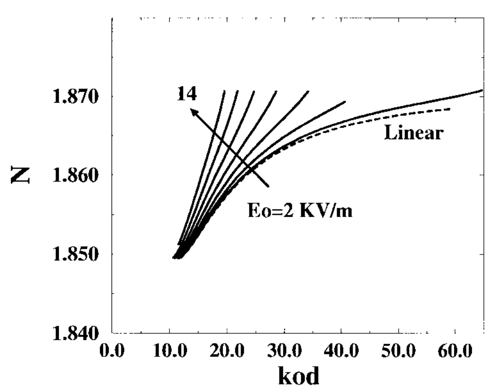

Fig. 4. The dispersion curve of the second TE-mode, TE_1 mode. $N = \beta/k_o$ ($\epsilon_{r1} = \epsilon_{r3} = 3.42, \epsilon_{r2} = 3.5$ and $\alpha > 0$).

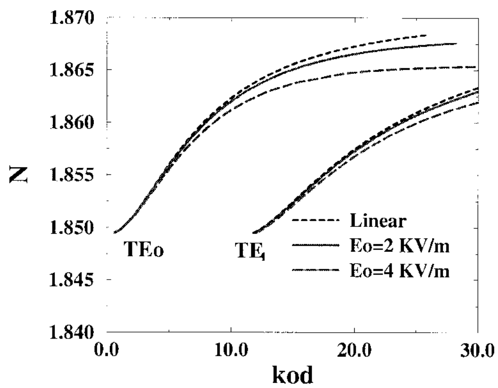

Fig. 5. The dispersion curves of TE_0 and TE_1 modes for the de-focusing nonlinearity. $\epsilon_{r2} = 3.5$, $\epsilon_{r1} = \epsilon_{r3} = 3.42$.

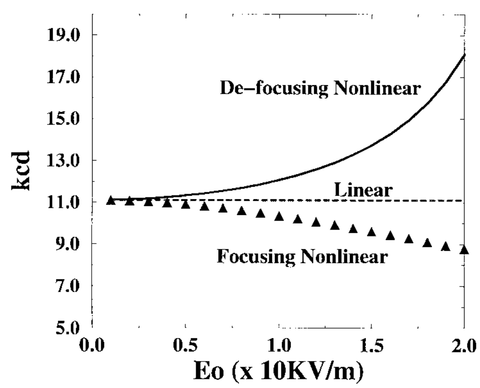

Fig. 6. The cutoff wavenumber of TE_1 versus E_o ($\epsilon_{r1} = \epsilon_{r3} = 3.42$, $\epsilon_{r2} = 3.5$).

respectively, for focusing nonlinearity. In Fig. 5 the dispersion curves of TE_0 and TE_1 modes for the defocusing nonlinearity are given.

In Figs. 6 and 7 the dependence of the cutoff wavenumber on E_o for TE_1 and TE_2 modes are shown for both focusing and de-focusing nonlinearities. From the figures it can be seen that the dispersion curves for the focusing nonlinear core are always above the linear dispersion curves and for the defocusing slab guide its dispersion curves are below the linear ones.

IV. Conclusion

In this paper, guided waves in nonlinear planar waveguides have been studied numerically. It is obvious from the results that the design of nonlinear guided waves strongly depends on the magnitude of the electrical field. The wave propagation properties illustrated here by numerical calculations are possibly applied to investigate devices composed of nonlinear planar waveguide structures.

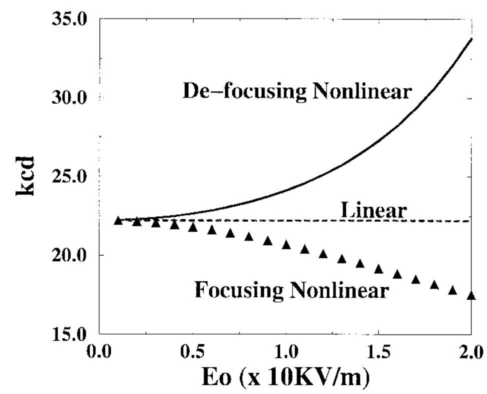

Fig. 7. The cutoff wavenumber of TE_2 versus E_o ($\epsilon_{r1} = \epsilon_{r3} = 3.42$, $\epsilon_{r2} = 3.5$).

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